THE ASYMPTOTIC LINES OF THE KUMMER SURFACE.

BY DR. J. I. HUTCHINSON.

(Read before the American Mathematical Society at the Meeting of April 29, 1899.)

THE asymptotic lines of the Kummer surface were first determined by Lie* by means of a transformation which establishes a correspondence between two spaces such that to the points of one space correspond the lines of a complex in the other. These curves have been the object of several other investigations,† all of which have treated the problem from the point of view of line geometry.

A very simple solution of the problem can be obtained without the aid of the line geometry by making use of the parametric representation of the Kummer surface in terms of hyperelliptic functions. Among the indefinite number of such representations that are possible, the one most convenient for the purpose is that given by Weber, by which the homogeneous coördinates of a point on the surface are taken proportional to four linearly independent theta functions of two variables u_1, u_2, of the second order and characteristic zero. The equation of a tangent plane can then be written‡

$$\vartheta(u+v)\,\vartheta(u-v)=0 \tag{1}$$

where $\vartheta(w)\,(=\vartheta(w_1,w_2))$ is a function of the first order and any characteristic, and v_1,v_2 are arbitrary constants. These latter may in fact be regarded as the tangential parameters for the Kummer surface in a sense the exact dual of that in which u_1,u_2 are the point parameters.

The two factors in (1) separately equated to zero represent the same curve. although discriminating between the two branches which pass through the point of contact of the

*S. Lie, "Sur une transformation géométrique," *Comptes rendus*, vol. 71 (1870), p. 579.

†Klein und Lie, "Ueber die Haupttangentencurven der Kummer'schen Fläche," *Monatsber. der Berl. Akad.*, 1870, p. 891; Reye, "Ueber die Singularitätenflächen, etc.," *Crelle*, vol. 97, p. 284; Segre, "Sur les courbes de tangents principales, etc.," *Crelle*, vol. 98, p. 301; Rohn, "Transformation der hyperelliptischen Functionen, etc.," *Math. Annalen*, vol. 15, p. 315.

‡Cf. Humbert, "Théorie générale, etc.," *Liouville's Journal*, 1893, p. 112.

tangent plane. The parameters for this point are the values of u_1, u_2 which satisfy the two equations

$$\begin{aligned} \vartheta(u+v) &= 0, \\ \vartheta(u-v) &= 0, \end{aligned} \tag{2}$$

simultaneously.

Let $u_1 + du_1$, $u_2 + du_2$ be the coördinates of a consecutive point on that branch of the curve represented by the equation $\vartheta(u+v) = 0$. We have then

$$\vartheta(u + du + v) = 0,$$

or, expanding in powers of du_1, du_2 and dropping infinitesimals of higher order.

$$\vartheta_1(u+v)du_1 + \vartheta_2(u+v)du_2 = 0, \tag{3}$$

where $$\vartheta_i(u+v) = \frac{\partial\vartheta(u+v)}{\partial(u_i+v_i)}.$$

The differential equation (3) gives one of the asymptotic directions through the point of tangency u_1, u_2.

Suppose $v_1 + dv_1$, $v_2 + dv_2$ are the coördinates of the tangent plane at the point $u_1 + du_1$, $u_2 + du_2$. Then for the new point we have a pair of simultaneous equations similar to (2), viz.:

$$\vartheta(u + du + v + dv) = 0,$$

$$\vartheta(u + du - v - dv) = 0.$$

From these we derive by expansion the two following:

$$\begin{aligned} &\vartheta_1(u+v)\cdot(du_1+dv_1) + \vartheta_2(u+v)\cdot(du_2+dv_2) = 0, \\ &\vartheta_1(u-v)\quad(du_1-dv_1) + \vartheta_2(u-v)\cdot(du_2-dv_2) = 0. \end{aligned} \tag{4}$$

Combining the first of these with equation (3) we have

$$\vartheta_1(u+v)\,(du_1-dv_1) + \vartheta_2(u+v)\,(du_2-dv_2) = 0. \tag{5}$$

Making this simultaneous with the second equation (4) we have either

$$\begin{vmatrix} \vartheta_1(u+v), & \vartheta_2(u+v) \\ \vartheta_1(u-v), & \vartheta_2(u-v) \end{vmatrix} = 0. \tag{6}$$

or

$$du_i - dv_i = 0. \qquad (i = 1, 2.) \tag{7}$$

Equation (6) determines those points whose asymptotic directions are coincident. It is, therefore, the equation of the parabolic curve. It is evident that if v_1, v_2 be a set of half periods, equation (6) is satisfied for all values of u_1, u_2 which satisfy (2). These determine the 16 conics of the Kummer surface, and as the combined degrees give exactly the right number 32 for the degree of the parabolic curve,* we have thus the complete solution of (6).

For the remaining points of the Kummer surface we have by integrating (7),

$$v_i = u_i + k_i, \qquad (i = 1, 2)$$

where k_1, k_2 are constants of integration. Substituting in (2) we have finally

$$\vartheta(2u + k) = 0, \tag{7}$$

$$\vartheta(k) = 0. \tag{8}$$

Equation (7)†, which should more properly be replaced by

$$\vartheta(2u + k)\,\vartheta(2u - k) = 0, \tag{10}$$

represents the ∞^1 asymptotic curves of the Kummer surface, the two constants k_i being connected by the relation (8). Equation (10) shows us at a glance that each curve of the family (evidently of degree 16) has a cusp at each node of the Kummer surface, and a double point at the place corresponding to the values of u_1, u_2 which annul simultaneously the two factors in the left member of (10

CORNELL UNIVERSITY,
March, 1899.

* See Klein und Lie, *loc. cit.*, p. 894.

† Cf. Reichardt, "Ueber die Darstellung der Kummer'schen Fläche, etc.," p. 479.

ON A DEFINITIVE PROPERTY OF THE COVARIANT.

BY MR. C. J. KEYSER.

(Read before the American Mathematical Society at the Meeting of April 29, 1890.)

THE general homogeneous entire polynomial of degree n in k variables x may be denoted by

$$F_n(x_1, x_2, \cdots, x_k) \equiv \Sigma c_{e_1 e_2 \cdots e_k} x_1^{e_1} x_2^{e_2} \cdots x_k^{e_k}$$

where $e_1 + e_2 + \cdots + e_k = n$. Let

$$\varphi_n(\xi_1, \xi_2, \cdots, \xi_k) \equiv \Sigma \gamma_{e_1 e_2 \cdots e_k} \xi_1^{e_1} \xi_2^{e_2} \cdots \xi_k^{e_k}$$

represent the polynomial into which F is converted by the substitutions

$$\begin{aligned} x_1 &= \lambda_{11}\xi_1 + \lambda_{12}\xi_2 + \cdots + \lambda_{1k}\xi_k, \\ x_2 &= \lambda_{21}\xi_1 + \lambda_{22}\xi_2 + \cdots + \lambda_{2k}\xi_k, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_k &= \lambda_{k1}\xi_1 + \lambda_{k2}\xi_2 + \cdots + \lambda_{kk}\xi_k, \end{aligned}$$

where the λ's are subject to a single restriction: their determinant D shall not assume the value zero.

If there is such an entire homogeneous polynomial

$$\psi_m(x_1, x_2, \cdots, x_k) \equiv \Sigma h_{e_1 e_2 \cdots e_k} x_1^{e_1} x_2^{e_2} \cdots x_k^{e_k},$$

where $e_1 + e_2 + \cdots + e_k = m$ and where each coefficient h is an entire homogeneous polynomial of degree p in the coefficients c of F, that

$$\psi_m(\xi_1, \xi_2, \cdots, \xi_k) \equiv M \cdot \psi_m(x_1, x_2, \cdots x_k),$$

the γ's entering the left member of the identity as the c's enter ψ_m of the right member, then $\psi_m(x_1, x_2, \cdots, x_k)$ is named covariant or invariant of F according as $m > 0$ or $= 0$.

Supposing such a function ψ to exist, it remains to determine the nature of the factor M. The ξ's and the γ's being linear respectively in the x's and the c's, the two members of the identity in question are, apart from the factor M, each of degree m in the x's and of degree p in the c's. It follows that M is a function of the λ's only. M is, more-

over, rational since ϕ is an entire polynomial and the equations of transformation are linear.

We may, then, write $M \equiv P_1 : P_2$, where the P's are homogeneous entire polynomials in the λ's and contain no common factor. Suppose first that M is not identically equal to a constant k'. If P_1 be not of the form $k_1 D^{\rho}$, then the λ's may be so chosen as to reduce P_1 to zero without causing either P_2 or D to vanish; for, if not, the locus $P_1 = 0$ would be a component of the locus $P_2 \cdot D = 0$, which is impossible inasmuch as D is not factorable and P_1 and P_2 have no factor in common. But if the λ's be chosen as indicated, $M = 0$ and consequently $\phi_m(\xi_1, \xi_2, \cdots, \xi_k)$ is seen to be identically zero, a result incompatible with the original supposition that the c's are entirely arbitrary. In like manner, if P_2 is not of the form $k_2 D^{\rho}$, it is possible by a suitable choice of the λ's to cause P_2 to vanish without reducing either P_1 or D to zero; but under this hypothesis covariants could not exist, for, on multiplying by P_2, the left member and hence the right member of the identity would be identically zero. Finally, if $M = k'$, then we may write $M = k' D^0$. Accordingly M must be of the form $k'' \cdot D^{\rho}$. By means of the transformation $x_1 = \xi_1$, $x_2 = \xi_2$, $\cdots$, $x_k = \xi_k$, it is readily found that $k'' = 1$. It thus appears that, under the definition, either no covariant exists or the factor M is of the form D^{ρ}.

In vol. I. of Jordan's Cours d'Analyse is found a proof of this proposition, in which the argument turns on the reversibility of the substitutions. A second demonstration, by Professor E. B. Elliott, occurs in vol. 16 of the *Messenger of Mathematics* and in his Introduction to the algebra of quantics. Here M is shown to be homogeneous, its degree in the λ's is determined, and then by help of the reversibility of the transformation a partial differential equation connecting M with its derivatives with respect to the λ's is obtained, whence the form of M is readily ascertained. In still a third proof by Professor Thomas S. Fiske, in vol. 19 of the journal just cited, the proposition defining the form of M is derived as a corollary from the converse there established of the multiplication theorem for determinants: A rational entire function having n^2 arguments and subject to the same law of multiplication as a determinant is a power of a determinant.

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THE KNOWN FINITE SIMPLE GROUPS.

BY PROFESSOR L. E. DICKSON.

(Read before the American Mathematical Society at the Meeting of April, 29, 1899.)

THE list of systems of simple groups given in the BULLETIN for May, 1897, may be enlarged by the addition of a number of new systems determined by the writer during the past two years. By request a table has been constructed which should aid in the determination of the status of a newly discovered simple group. The first part of the table gives the orders of all simple groups of order less than one million which are included in the various systems except the first. In extending the table to the orders between one million and one billion, I have excluded, for the sake of brevity, the simple groups $LF(2,p^n)$ of type (3) having the orders

$$(p^{2n}-1)p^n \quad \text{for } p=2,$$

$$\tfrac{1}{2}(p^{2n}-1)p^n \quad \text{for } p>2.$$

But it is readily determined whether or not a given number N is of one of these two forms. Indeed, if q^r be the largest power of a prime contained in N, it must equal p^n or p^n+1. Hence, if q^r-1 be a power of a prime. then $p^n=q^r-1$; in the contrary case, $p^n=q^r$.

The calculations have all been made in duplicate; furthermore, care has been taken to include all orders coming within the assigned limits of the table.

In the following systems of simple groups, p denotes a prime number, m and n denote integers, each subject only to the limitations given.

(1) $$p,$$

(2) $$\tfrac{1}{2}n! \quad (n>4),$$

(3) $$LF(m,p^n)\equiv\frac{1}{d}(p^{nm}-1)p^{n(m-1)}(p^{n(m-1)}-1)p^{n(m-2)} \cdots (p^{2n}-1)p^n$$

where $p^n>3$ if $m=2$. Here d denotes the greatest common divisor of m and p^n-1.

$$HO(m, p^{2n}) \equiv \frac{1}{g}\,[p^{nm} - (-1)^m]\,p^{n(m-1)}\,[p^{n(m-1)} - (-1)^{m-1}]\,p^{n(m-2)} \cdots [p^{2n} - 1]p^n \tag{4}$$

where $p^{2n} > 3^2$ if $m = 2$ and $p^{2n} + 2^2$ if $m = 3$. Here g denotes the greatest common divisor of m and $p^n + 1$.

$$A(2m, p^n) \equiv \frac{1}{a}(p^{2nm} - 1)p^{n(2m-1)}(p^{n(2m-2)} - 1)p^{n(2m-3)} \cdots (p^{2n} - 1)p^n \tag{5}$$

where $p^n > 3$ if $m = 1$ and $p^n + 2$ if $m = 2$. Here $a = 1$ if $p = 2$ and $a = 2$ if $p > 2$.

$$FH(2m, 2^n) \equiv (2^{nm} - 1)[(2^{2n(m-1)} - 1)2^{2n(m-1)}][(2^{2n(m-2)} - 1)2^{2n(m-2)}] \cdots [(2^{2n} - 1)2^{2n}] \tag{6}$$

where $m > 2$.

$$SH(2m, 2^n) \equiv (2^{nm} + 1)[(2^{2n(m-1)} - 1)2^{2n(m-1)}] \cdots [(2^{2n} - 1)2^{2n}] \tag{7}$$

where $m > 1$.

$$O(2m + 1, p^n) \equiv \tfrac{1}{2}(p^{2nm} - 1)p^{n(2m-1)}(p^{n(2m-2)} - 1)p^{n(2m-3)} \cdots (p^{2n} - 1)p^n \tag{8}$$

where $p > 2$ and where $p^n + 3$ if $2m + 1 = 3$.

$$O(2m, p^n) \equiv \tfrac{1}{2}[p^{n(2m-1)} - \epsilon^m p^{n(m-1)}](p^{n(2m-2)} - 1)p^{n(2m-3)} \cdots (p^{2n} - 1)p^n \tag{9}$$

where $p > 2$ and $2m > 4$.

$$NS(2m, p^n) \equiv \tfrac{1}{2}[p^{n(2m-1)} + \epsilon^m p^{n(m-1)}](p^{n(2m-2)} - 1)p^{n(2m-3)} \cdots (p^{2n} - 1)p^n \tag{10}$$

where $p > 2$ and $2m > 2$.

In (9) and (10), $\epsilon = \pm 1$ according as p^n is of the form $4l \pm 1$.

The group (3) is obtained* from the decomposition of the general linear homogeneous group on m indices in the $GF[p^n]$; concretely, it is the linear fractional group of determinant unity. The largest linear homogeneous group in m variables in the $GF[p^{2n}]$ leaving invariant

* Writer's Dissertation, *Annals of Mathematics*, 1897.

$$\xi_1 x^{p^s+1} + \cdots + \xi_m x^{p^s+1}$$

may be called the *hyperorthogonal* group, since it has as a subgroup the general orthogonal group on m indices in the $GF[p^s]$. Its decomposition leads * to the simple group (4). The group (5) is obtained† from the abelian group on $2m$ indices in the $GF[p^s]$. Groups (6) and (7) are subgroups of index two under the generalized first‡ and second§ hypoabelian groups respectively; viz., linear groups on $2m$ indices in the $GF[2^s]$. Groups (8) and (9) are obtained from the decomposition of the orthogonal group in the $GF[p^s]$. Group (10) is obtained from the group defined by the invariant

$$\sum_{i=1}^{2m-1} \xi_i^2 \xi_{i+1}^2 + \nu \xi_{2m}^2$$

where ν is a not-square in the $GF[p^s]$. For the last three groups I refer to the memoir recently presented to the *American Journal of Mathematics*. Aside from certain low cases the results for the groups (8) and (9) had been previously‖ determined by the writer employing different methods.

Between certain groups given by the above systems there exist simple isomorphisms. The group $A(2, p^{ns})$ is identical with $LF(2, p^s)$. The groups $HO(2, p^{2n})$ and $O(3, p^s)$ are isomorphic with $LF(2, p^s)$. Indeed, the first is the so-called "imaginary form" of the third. We will, therefore, not mention $A(2, p^s)$ and $HO(2, p^{2n})$ in our tables. $SH(4, 2^s)$ is isomorphic with $LF(2, 2^{2s})$; $NS(4, p^s)$ with $LF(2, p^{2s})$ if $p \pm 2$. According as p^s is of the form 2^s, $4l+1$, or $4l-1$, the simple group designated by $LF(4, p^s)$ is simply isomorphic with the simple group designated by $FH(6, 2^s)$, $O(6, p^s)$, or $NS(6, p^s)$, respectively. I have not yet completed an investigation which promises to show that, according as $p^s = 2^s$, $4l+1$, $4l-1$, the group $HO(4, p^{2s})$ is simply isomorphic with the group $SH(6, 2^s)$, $NS(6, p^s)$, $O(6, p^s)$, respectively. I have succeeded in proving that the groups $HO(4, 2^2)$ and $SH(6, 2)$ are simply isomorphic, and have set up an abstract group and a substitution group on 36 letters simply isomorphic to each of the two linear groups. As a special case of the result referred

* The paper proving this result was offered January 22 to the *Math. Annalen*.

† *Quar. Jour. of Math.*, No. 114, 1897.

‡ *Quar. Jour. of Math.*, No. 117, 1898; BULLETIN, July, 1898.

§ "Determination of the structure of all linear homogeneous groups in a Galois field which are defined by a quadratic invariant," offered January 3d to the *Amer. Jour. of Math.*

‖ BULLETIN, February and May, 1898; *Proc. of the Calif. Acad. of Sciences*, third series, vol. 1, Nos. 4 and 5.

to below, the groups $O(5, 3)$ and $A(4, 3)$ are simply isomorphic. We have, therefore, four simple groups of order 25920, simply isomorphic in sets of two. [In a paper presented March 9, before the London Mathematical Society, I have shown that all four groups are simply isomorphic.]

Of the four known simple groups of order 20160, it was noted above that $LF(4, 2)$ and $FH(6, 2)$ are isomorphic.* Jordan has proven† that $LF(4, 2)$ is simply isomorphic to the alternating group on 8 letters. A very elegant group-theoretic proof has been recently given by Moore, "Concerning the general equations of the seventh and eighth degrees," *Mathematische Annalen*, vol. 51, pp. 417–444, in particular pp. 435–6. Finally, the alternating group on 8 letters is *not* isomorphic to the group $LF(3, 2^2)$, proven simple in the writer's Dissertation. This result (not yet published) has been reached by Miss Schottenfels, of Chicago, under the direction of Professor Moore.

The groups $O(5, p^n)$ and $A(4, p^n)$ are simply isomorphic.‡ Although the simple groups (8) and (5), for $p > 2$, have the same order, it seems probable that they are isomorphic only when $m = 1$ and 2; indeed, the corresponding continuous groups are not isomorphic if $m > 2$.

Aside from the cases just discussed, no coincidence in the orders of the simple groups in the above ten systems has been observed by the writer. Moreover, for the sets of values of m, n, p excluded in the above systems, there occur no simple groups of composite order in the series of factor groups for the corresponding linear groups, exceptions to this statement being the first hypoabelian group on $2m = 4$ indices and the orthogonal group on $2m = 4$ indices and the abelian group on 4 indices modulo 2. In the first of these three cases, the group has two of its factor groups identical with $LF(2, 2^n)$; in the second case, the group has two of its factor groups identical with $LF[2, p^n]$, $p > 2$; in the third case, the group is isomorphic to the alternating group on six letters.§

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* The general theorem, of which the result quoted is a special case, is given in the paper, "The structure of certain linear groups with quadratic invariants," communicated first to Professor Moore, May 11, 1898, and received for publication by the London Mathematical Society, September 1, 1898.

† Traité des Substitutions, No. 516.

‡ Dickson, "Concerning the abelian and hypoabelian groups," BULLETIN, p. 332, April, 1899; to be published in full in the Transactions of the Society.

§ Jordan, Traité des Substitutions, § 335. The writer has recently found a simple group theoretic proof, based on a correspondence of the generators and generational relations of the two groups.

In the following table of simple groups, a known simple isomorphism between two groups is indicated by the equality sign, a known non-isomorphism by the inequality sign.

Order of simple groups.	Character of simple groups.
60	$2^2.3.5$	$LF(2,2^2)=LF(2,5)=O(3,5)=SH(4,2)=\frac{1}{2}5!$
168	$2^3.3.7$	$LF(2,7)=O(3,7)=LF(3,2)$
360	$2^3.3^2.5$	$LF(2,3^2)=O(3,3^2)=NS(4,3)=\frac{1}{2}6!$
504	$2^3.3^2.7$	$LF(2,2^3)$
660	$2^2.3.5.11$	$LF(2,11)=O(3,11)$
1 092	$2^2.3.7.13$	$LF(2,13)=O(3,13)$
2 448	$2^4.3^2.17$	$LF(2,17)=?(3,17)$
2 520	$2^3.3^2.5.7$	$\frac{1}{2}7!$
3 420	$2^2.3^2.5.19$	$LF(2,19)=O(3,19)$
4 080	$2^4.3.5.17$	$LF(2,2^4)=SH(4,2^4)$
5 616	$2^4.3^3.13$	$LF(3,3)$
6 048	$2^5.3^3.7$	$HO(3,3^2)$
6 072	$2^3.3.11.23$	$LF(2,23)=O(3,23)$
7 800	$2^3.3.5^2.13$	$LF(2,5^2)=O(3,5^2)=NS(4,5)$
7 920	$2^4.3^2.5.11$	Cole, *Quarterly Journal*, vol. 27, p. 48.
9 828	$2^2.3^3.7.13$	$LF(2,3^3)=O(3,3^3)$
12 180	$2^2.3.5.7.29$	$LF(2,29)=O(3,29)$
14 880	$2^5.3.5.31$	$LF(2,31)=O(3,31)$
20 160	$2^6.3^2.5.7$	$\frac{1}{2}8!=LF(4,2)=FH(6,2)+LF(3,2^2)$
25 308	$2^2.3^2.19.37$	$LF(2,37)=O(3,37)$
25 920	$2^6.3^4.5$	$A(4,3)=O(5,3)=SH(6,2)=HO(4,2^2)$
32 736	$2^5.3.11.31$	$LF(2,2^5)$
34 440	$2^3.3.5.7.41$	$LF(2,41)=O(3,41)$
39 732	$2^2.3.7.11.43$	$LF(2,43)=O(3,43)$
51 888	$2^4.3.23.47$	$LF(2,47)=O(3,47)$
58 800	$2^4.3.5^2.7^2$	$LF(2,7^2)=O(3,7^2)=NS(4,7)$
62 400	$2^6.3.5^2.13$	$HO(3,2^4)$
74 412	$2^2.3^3.13.53$	$LF(2,53)=O(3,53)$
95 040	$2^6.3^3.5.11$	Substitution Group (Mathieu, Miller)
102 660	$2^2.3.5.29.59$	$LF(2,59)=O(3,59)$
113 460	$2^2.3.5.31.61$	$LF(2,61)=O(3,61)$
126 000	$2^4.3^2.5^3.7$	$HO(3,5^2)$
150 348	$2^2.3.11.17.67$	$LF(2,67)=O(3,67)$
178 920	$2^3.3^2.5.7.71$	$LF(2,71)=O(3,71)$
181 440	$2^6.3^4.5.7$	$\frac{1}{2}9!$
194 472	$2^3.3^3.37.73$	$LF(2,73)=O(3,73)$
246 480	$2^4.3.5.13.79$	$LF(2,79)=O(3,79)$
262 080	$2^6.3^2.5.7.13$	$LF(2,2^6)=SH(4,2^3)$
265 680	$2^4.3^4.5.41$	$LF(2,3^4)=O(3,3^4)=NS(4,3^2)$
285 852	$2^2.3.7.41.83$	$LF(2,83)=O(3,83)$
352 440	$2^3.3^2.5.11.89$	$LF(2,89)=O(3,89)$
372 000	$2^5.3.5^3.31$	$LF(3,5)$
456 288	$2^5.3.7^2.97$	$LF(2,97)=O(3,97)$
515 100	$2^2.3.5^2.17.101$	$LF(2,101)=O(3,101)$
546 312	$2^3.3.13.17.103$	$LF(2,103)=O(3,103)$
612 468	$2^2.3^2.53.107$	$LF(2,107)=O(3,107)$
647 460	$2^2.3^2.5.11.109$	$LF(2,109)=O(3,109)$
721 392	$2^4.3.7.19.113$	$LF(2,113)=O(3,113)$
985 720	$2^3.3.5.11^2.61$	$LF(2,11^2)=O(3,11^2)=NS(4,11)$

Order of simple groups.		Character of simple groups.
976 500	$2^2.3^2.5^3.7.31$	$LF(2,5^3) = O(3,5^3)$
979 200	$2^5.3^2.5^2.17$	$A(4,2^2)$
1 451 520	$2^9.3^4.5.7$	$A(6,2)$
1 814 400	$2^7.3^4.5^2.7$	$\frac{1}{2}10\,!$
1 876 896	$2^5.3^2.7^3.19$	$LF(3,7)$
3 265 920	$2^7.3^5.5.7$	$HO(4,3^2)$, $O(6,3)$
4 680 000	$2^6.3^2.5^4.13$	$A(4,5)$
5 515 776	$2^9.3^4.7.19$	$HO(3,2^6)$
5 663 616	$2^7.3.7^3.43$	$HO(3,7^3)$
6 065 280	$2^7.3^5.5.13$	$LF(4,3)$
9 999 360	$2^{10}.3^2.5.7.31$	$LF(5,2)$
13 685 760	$2^{10}.3^5.5.11$	$HO(5,2^2)$
16 482 816	$2^9.3^2.7^2.73$	$LF(3,2^3)$
19 958 400	$2^7.3^4.5^2.7.11$	$\frac{1}{4}11\,!$
42 456 960	$2^7.3^6.5.7.13$	$LF(3,3^2)$
42 573 600	$2^5.3^5.5^2.73$	$HO(3,3^4)$
70 915 680	$2^5.3^2.5.11^3.37$	$HO(3,11^2)$
138 297 600	$2^9.3^2.5^2.7^4$	$A(4,7)$
174 182 400	$2^{12}.3^5.5^2.7$	$FH(8,2)$
197 406 720	$2^{13}.3^4.5.7.17$	$SH(8,2)$
212 427 600	$2^4.3.5^2.7.11^3.19$	$LF(3,11)$
239 500 800	$2^9.3^5.5^2.7.11$	$\frac{1}{2}12\,!$
270 178 272	$2^5.3^4.7.13^3.61$	$LF(3,13)$
811 273 008	$2^4.3.7^3.13^3.157$	$HO(3,13^2)$
987 033 600	$2^{13}.3^4.5^2.7.17$	$LF(4,2^2) = FH(6,2^2)$

I added a few simple groups of order greater than a billion, but following immediately those given in the table.

$LF(4,5) = 2^7.3^2.5^6.13.31$, $LF(4,3^2) = 2^{10}.3^{12}.5^2.7.13.41$,
$LF(5,3) = 2^9.3^{10}.5.11^2.13$, $LF(5,2^2) = 2^{20}.3^5.5^2.7.11.17.31$,
$LF(5,5) = 2^{11}.3^2.5^{10}.7.13.31.61$, $LF(6,2) = 2^{15}.3^4.5.7^2.13$,
$HO(4,2^4) = 2^{12}.3^4.5^3.13.17$, $HO(4,5^2) = 2^7.3^4.5^6.7.13$,
$HO(5,3^2) = 2^{11}.3^{10}.5.7.61$, $HO(6,2^2) = 2^{15}.3^7.5.7.11$,
$A(4,2^3) = 2^{12}.3^4.5.7^2.13$, $A(4,3^2) = 2^8.3^6.5^2.41$,
$A(4,11) = 2^5.3^2.5^2.11^4.61$, $A(6,3) = 2^9.3^5.5.7.13$,
$A(6,2^2) = 2^{16}.3^4.5^3.7.13.17$, $A(6,5) = 2^9.3^4.5^3.7.13.31$,
$A(8,2) = 2^{16}.3^5.5^2.7.17$, $A(8,3) = 2^{14}.3^{16}.5^2.7.13.41$,
$FH(6,2^3) = 2^{15}.3^4.5.7^3.13.73$, $FH(8,2^2) = 2^{24}.3^5.5^4.7.13.17^2$,
$FH(10,2) = 2^{20}.3^5.5^2.7.17.31$,
$SH(6,2^4) = 2^{12}.3^2.5^3.13.17$, $SH(10,2) = 2^{20}.3^5.5^2.7.11.17$,
$NS(6,5) = 2^7.3^4.5^6.7.13$, $NS(6,3^2) = 2^8.3^{12}.5^3.41.73$,
$NS(8,3) = 2^{10}.3^{12}.5.7.13.41$, $NS(8,5) = 2^{10}.3^4.5^{12}.7.13.31.313$,
$O(6,7) = 2^{10}.3^2.5^4.7^6.43$, $O(6,11) = 2^7.3^4.5^7.11^6.37.61$,
$O(8,3) = 2^{18}.3^{12}.5^2.7.13$, $O(8,5) = 2^{12}.3^5.5^{12}.7.13^2.31$.

UNIVERSITY OF CALIFORNIA,
February 1, 1899.

THE GEOMETRY OF MOVEMENT.

Geometrie der Bewegung in synthetischer Darstellung. Von Dr. ARTHUR SCHOENFLIES. Leipzig, B. G. Teubner, 1886. 8vo, pp. vi + 194.

La Géométrie du Mouvement. Exposé synthétique. Translated by CH. SPECKEL, Capitaine du Génie. Paris, Gauthier-Villars, 1893. 8vo, pp. vii + 292.

PERHAPS apology is needed for noticing a book no longer in its infancy. But we feel that "better late than never" applies to acquaintance with a work which contains so much matter which was new at the time of writing and is not yet accessible in English, nor (we believe) well known.

The main idea of the book is to consider a body in two or more positions relatively to another body, and thence as a limit case to discuss the instantaneous motion. Full advantage is taken of the duality arising from viewing things from the standpoint of the one body or the other. That we have found the book difficult is probably due to our early training; but a few more figures and a few more details would have been welcome.

The French translation is very reliable, and its value is increased by a good elementary account of complexes and congruences of lines (pp. 219–291). by G. Fouret.

Our intention is, not to discuss the information in the book, but to select a few of the more salient theorems (omitting such as are presumably familiar). When in this string of enunciations there seems occasion to interpolate a remark, square brackets are used.

§1. *Motion of a Plane in a Plane.*

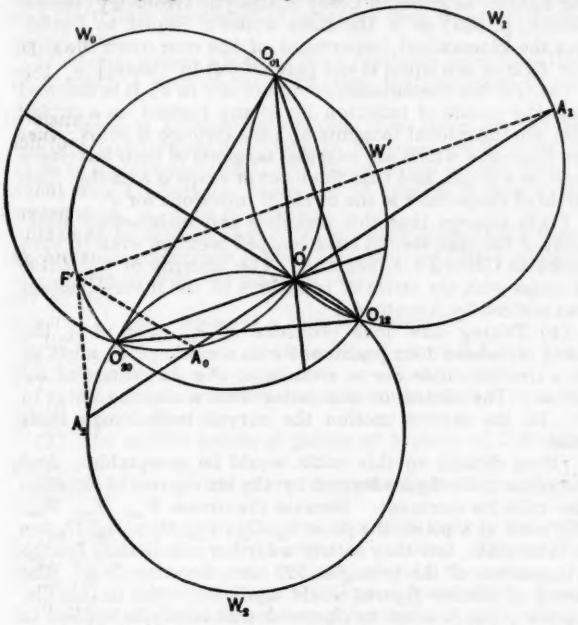
(1) Let σ_0, σ_1, σ_2 be three positions of a plane σ, relative to a plane σ'. Let A_0, A_1, A_2 be the three positions of a point A of σ, A' the point of σ' equidistant from A_0, A_1, A_2. The correspondence of the points A and A' is quadratic; to a line in the one plane corresponds a conic in the other, passing through the three centres of rotation in that other.

(2) All the lines of σ which for the three positions meet in a point of σ' form a pencil in σ. All the points of σ which lie on a line of σ' lie on the circumcircle of the centres of rotation in σ.

This circle is called proleptically the "circle of inflexions."

[Two questions arise here which are not considered in the book: What is the vertex of the pencil in σ, and what is

the envelope of the lines in σ'? To answer these consider the accompanying kaleidoscopic figure.



In the figure O_{01}', O_{12}', O_{20}', are the centres of rotation about which σ is brought from the position σ_0 into σ_1, σ_1 into σ_2, σ_2 into σ_0. O' is the orthocentre of the three centres of rotation, and the four equal circles are the circles of inflexion. We may replace the rotations by pairs of reflexions;* σ_0, σ_1, and σ_2 are simply reflexions of σ' in the sides of the triangle.

The reflexions of a point F' on the circle W' lie on a line (the directrix of the inscribed parabola whose focus is F') which passes through O'. Hence:

When a point of σ, in its three positions, lies on a line of σ', that line passes through the orthocentre O'.

And hence, considering the inverse motion, when a line of σ', in three positions, passes through a point of σ, the line must pass through the orthocentre of the triangle of rotations in σ'.

*Elsewhere also in the book the notion of reflexion might have been useful.

These facts are implicitly contained in the theory of similar figures, as given in Casey's Analytic Geometry (second edition, p. 395) or in the same writer's Sequel to Euclid. But the kinematical importance of the case when the similar figures are equal is not pointed out by Casey.]

(3) For the instantaneous motion of σ in σ', it is deduced that the points of inflexion lie at any instant on a circle; that the inflexional tangents all pass through a point; that the lines of σ which are cuspidal tangents of their envelopes meet in a point, and that the locus of cusps is a circle. This circle of cusps for σ is the circle of inflexions for σ'.

[It is strange that this complete and satisfactory statement of the case should have escaped mention even in such books as Clifford's Kinematic. The identity of the circle of cusps with the circle of inflexions of the inverse motion was noticed by Aronhold.]

(4) Taking now four positions σ_0, σ_1, σ_2, σ_3 of σ, the point A, whose four positions lie on a circle of σ', must lie on a circular cubic curve, containing the six centres of rotation. The centres of the circles form a circular cubic in σ'. In the inverse motion the curves interchange their rôles.

[More details on this cubic would be acceptable. And the remarkable figure formed by the six centres of rotation also calls for comment. Because the circles W_{123}, W_{230}, W_{301}, W_{012} meet at a point, the pairs O_{01}, O_{23}; O_{02}, O_{31}; O_{03}, O_{12} are in involution, but they satisfy a further condition; for the orthocentres of the triangles 123 etc., lie on a line. The theory of similar figures would again be useful in this discussion; but it must be observed that it can be applied in eight different ways.]

(5) In *five* positions of σ there are in general 4 points which in the 5 positions lie on a circle.

§2. *Motion of a Body about a fixed Point.*

(1) In two positions S_0 and S_1 in a space S corresponding planes of two pencils meet on an orthogonal cone (*i. e.*, on a cone of which a circular section is perpendicular to a generator).

Hence, in the instantaneous motion, the planes of a pencil touch their envelopes in lines which form an orthogonal cone. One set of circular sections is perpendicular to the axis of the pencil, the other to the instantaneous axis.

For three positions we have, as before, a quadratic correspondence between a moving ray a and the axis of the circular cone of S through the 3 positions of a.

(2) Rays which in the 3 positions lie in a plane of S' form in S a cubic cone containing the 3 instantaneous axes and the 6 lines of contact of tangents from these to the absolute cone.

Inversely, planes which in the 3 positions meet in a line of S' touch a cone of the third class, the reciprocal of the other.

[Stripped of kinematical notions, these cones are the locus of a point whose reflexions in 3 planes which meet at O lie in a plane through O, and the envelope of that plane.]

(3) For 4 positions, there are in general 6 rays a for which a_0, a_1, a_2, a_3 lie in a plane.

[It will be noticed that cones are used throughout these theorems in preference to their intersections with a concentric sphere or with the plane at infinity.]

§3. *Motion of a Body Σ in Space Σ'.*

(1) In two positions, the middle points of the two positions of points of a line g of Σ lie on a line g'' of Σ'.

(2) The displacement of the line can be effected by a rotation about a line g^r.

(3) The middle points of points of a plane of Σ lie in a plane of Σ'.

(4) The lines g'' and g^r are conjugate lines of a null-system.

(5) The most general motion of a rigid body is the rolling and sliding of a ruled surface on another. The parameters, for the generators in contact, must be the same.

[We are not repeating a well known proposition here, for the last condition is not brought out clearly as a rule.]

(6) The tangents of the trajectories of all the points of a body, at any instant, form a tetrahedral complex. The fundamental tetrahedron reduces to the axis of rotation and the line at infinity in a perpendicular plane.

(7) The characteristics of a pencil of planes form at each instant, in general, an orthogonal hyperboloid (*i. e.*, a hyperboloid of which a set of circular sections is perpendicular to a generator).

(8) The lines of the complex which pass through a point form an orthogonal cone.

(9) All points of Σ which are aiming at a point of Σ' lie on a circular cubic curve in space, the partial intersection of a right cylinder and an orthogonal cone. The axis of rotation is the real asymptote of the cubic.

[This pretty cubic can be very simply constructed by

drawing the curve $y = \tan\frac{x}{2}$ and rolling the paper into a cylinder of radius 1, so that the asymptotes coincide.]

(10) The characteristics of all planes which pass through a point are at any instant secants of a circular cubic space curve.

(11) All points of Σ which, in three positions, lie in a line of Σ' form a cubic space curve through the points at infinity on the 3 axes of displacement.

(12) All points of Σ which in 4 positions lie in a plane of Σ' form a cubic surface.

(13) All points of Σ which in 5 positions lie in a plane form a sextic space curve.

(14) There are in general 10 points which in 6 positions lie in a plane.

(15) The points which in 5 positions lie on a sphere form a quartic surface. The centres of the spheres form a quartic in Σ'. In the inverse movement the quartics interchange their rôles.

(16) The points which in 6 positions lie on a sphere form a space curve of order 10.

(17) There are in general 20 points which in 7 positions lie on a sphere.

F. MORLEY.

HAVERFORD COLLEGE.

THE NEW EDITION OF WEBER'S ALGEBRA.

Lehrbuch der Algebra. By HEINRICH WEBER. Second edition. Braunschweig, Vieweg und Sohn, 1898–99. 8vo. Vol. I, pp. 703. Vol. II, pp. 855.

MOST of the readers of the BULLETIN have been aware that a new edition of Weber's Algebra was in progress. Some time ago the first volume appeared; the work is now complete, the second volume of the new edition having just come out. The many excellencies of this great work have been so generally appreciated that Professor Weber has experienced the very unusual pleasure of seeing a new edition required in less than two years after the publication of the first.

The first question that those who already have purchased the first edition will wish answered is: Are the changes so great as to make it desirable to secure the new edition? To

this we would reply: University and seminary libraries and all those who are especially interested in algebraical speculations should not fail to have the new edition. For such it will be comforting to know that the price of the new edition is reduced by almost half. On the other hand, we do not consider it necessary for those who have only an occasional or incidental interest in algebra to put themselves to the expense of purchasing new copies.

We turn now to indicate rapidly some of the more important changes. The first thing that strikes the reader is that the work has increased considerably in size, in spite of the manifest effort of its author to keep it within bounds. The first volume has increased 50 pages, the second 61 pages, so that the work has now 1558 pages. The task of mastering such bulky volumes may well at first intimidate the prospective reader. He will, however, generally be interested in only some of the many subjects treated; the remainder will form a convenient store for reference. The size is, thus, no particular objection, since the work is so arranged that it is not necessary to read it consecutively.

The next feature that one observes in comparing carefully the two editions is the great number of slight changes that have been made. Certain paragraphs have been rewritten; others left out or transposed; still others are wholly new. The whole work has been subjected to careful scrutiny and no pains have been spared to make the style clearer and the demonstrations simpler or more rigorous. The cumulative effect of all this is to enhance the value of the work very materially.

Let us illustrate this by looking over chapter XVI on cyclotomy. It opens by discussing the irreducibility of the polynomials X_n. In the old edition this was considered in chapter XII when treating of the roots of unity. The form of the proof is changed; in the old edition a proof of Kronecker was followed, here one by Dedekind is given. In discussing the sums of Gauss Σr^a, Σr^b, the new edition follows Kronecker, while the old followed Gauss. Finally a new paragraph on the polynomials

$$\prod_a (x - r^a), \qquad \prod_b (x - r^b)$$

is inserted. Such changes run more or less throughout the book.

Let us note now some of the changes of greater extent. In volume I these are limited to two chapters, the fourth and the eighth. In chapter IV the theory of elimination has

been treated with much more detail and this is a most welcome addition. The additions of chapter VIII incorporate some of Hurwitz's researches relating to Sturm's theorem.

In volume II the first important addition we meet is an account of Dedekind's researches on Hamiltonian groups in chapter IV. Chapter VI has been completely remodeled; the part relating to Klein's Formenproblem and the invariants of linear substitution groups has been thrown into a new chapter and in its place we find the first properties of matrices given. The theory of matrices which was not mentioned in the old edition has been inserted in order to give an account of Frobenius's remarkable papers on groups which have appeared recently in the *Sitzungsberichte of the Berlin Academy*. These treat of the characters and determinants of finite groups and their representation by linear substitutions. The fourth book, which treats of algebraic numbers, has been altered more than any other part of the whole work; still its general character has been maintained. We call attention to a number of points: First, the insertion of Kronecker's method of breaking up a form into its irreducible factors for a given domain. A more extensive use of Minkowski's ideas relating to the geometry of numbers has been made. In the present edition a special chapter is devoted to them. The chapter on the relation of a body to its divisors gives a somewhat fuller account of Hilbert's researches. Finally, a remarkable application due to Dedekind of the theory of the Classenzahl is made to prove Legendre's postulate that every arithmetic progression

$$ax + b$$

where a and b are integers and relative primes contains an indefinite number of primes.

We rejoice to learn, by remarks made in two places, that the author still contemplates writing a third volume, which is destined to treat of the application of the general theory of algebraic numbers to the complex multiplication of elliptic functions. We are sure we express the unanimous sentiment of the whole mathematical community in hoping that Professor Weber will resolutely persevere in this arduous undertaking.

JAMES PIERPONT.

YALE UNIVERSITY,
May, 1899.

SHORTER NOTICES.

Corso de Analyse Infinitesimal. F. GOMES TEIXEIRA. 3 vols., 8vo. Porto, 1892–96.

WHILE perusing the present book it was a constant source of regret to me that Portuguese is not better known in our country. Otherwise this admirable work on the calculus would enjoy widespread popularity among us. Its author, the distinguished director of the Academia Polytechnica at Porto, has been uniformly successful in the difficult task of selecting from the immense material available. The manner of presentation leaves nothing to be desired. The style is lucid and elegant, and the whole work bears in a refreshing manner the imprint of an original mind. In many places the author has incorporated parts of his own prolific and valuable writing on the subject. In regard to rigor, it seems to us that Professor Teixeira has very happily chosen the golden mean. The excessive rigor of a Weierstrassian has been wisely avoided; at the same time the author has given this matter due attention. An occasional slip will doubtless be corrected in later editions. Altogether the work has so favorably impressed us that we should prefer to see it translated into English rather than any other work on the subject we know of. It is a deplorable confession that the English language does not to-day possess a work on the calculus of this class. We indicate very briefly its contents.

Differential Calculus, pp. 383.—The first hundred pages are devoted to a careful exposition of the fundamental notions and processes of analysis; numbers, limits, infinite series and products, continued fractions, continuity. At page 112 the differential calculus proper begins. We touch only on one or two matters. Th esubject of implicit functions, ordinarily treated so carelessly, is given here with rigor and elegance. In this connection the elementary properties of functional determinants are given. The exact form of higher derivatives is given with more than usual fullness. Application is made to Bernoulli's numbers and formulæ of Waring. The treatment of maxima and minima of functions of two variables is incorrect. The error which the writer of these lines criticised in the BULLETIN of July, 1898, has also been made in this place. After the usual application of Taylor's series to geometry follows an interesting chapter on functions defined by series.

Hankel's principal of condensation of singularities is given, and functions without derivatives are illustrated by Weierstrass's classic example. The volume closes with a remarkably rich chapter on functions of a complex variable. Weierstrass's and Mittag-Leffler's theorems are given, and existence of lacunary functions illustrated.

Integral Calculus, Part I., pp. 339.—The first half of the volume is devoted to the usual topics. We notice as praiseworthy in particular the author's treatment of the reduction of hyperelliptic integrals, the theorems of Green, Cauchy, and Stokes. Among the applications of definite integrals we mention Gauss's proof of the fundamental theorem of algebra, Sturm's theorem, and Fourier's series.

The second half of this volume contains a good elementary account of differential equations. Without giving the reader the disagreeable feeling that unduly much has been forcibly compressed into a very small space, a sufficient orientation is given not only in ordinary but also in partial differential equations of the first and second orders.

Integral Calculus, Part II., pp. 348.—The first eighty pages treat of Cauchy's theory of integrals between complex limits with application to the calculation of definite integrals and to the development of functions into power series, into series of rational functions, into Fourier's series, and into series according to the powers of sine and cosine.

About 125 pages are devoted to the elliptic functions. The starting point is the definite integral, first for real, then for imaginary, and finally by virtue of the addition theorem for complex argument. From a second point of view they are defined as infinite series in terms of their poles. The development of the sigmas in Fourier's series makes it necessary to multiply them by exponential factors. This leads very naturally to the thetas and the relations existing between these two classes of functions. Instead now of studying the elliptic functions of Jacobi by means of the thetas, the author prefers to do this through $\wp$ and σ (by means of the integral definition). The reader gains a good idea of the way to use the elliptic functions by well chosen applications to geometry. A chapter on many valued functions and a sketch of the elements of calculus of variations terminate this excellent work.

JAMES PIERPONT.

Verhandlungen des ersten internationalen Mathematiker-Kongresses in Zürich vom 9 bis 11 August, 1897. Herausgegeben von FERDINAND RUDIO. Leipzig, B. G. Teubner, 1898. 8vo, viii + 306 pp.

THE Zürich congress will always possess an especial interest as the first international congress. The present stately octavo volume is the *official* report of the same, prepared by Professor Rudio, one of the two general secretaries.

The first eighty pages recount in an agreeable manner the events of the congress and are very interesting reading. The address of welcome by the president, Professor Geiser, and the paper by Professor Rudio "On the object and organization of international congresses" are given in full. Professor Geiser made felicitous reference to the great names of Jacob, Daniel, and Johann Bernoulli, and of Euler and Steiner as mathematicians of the past of whom Switzerland was justly proud. Their portraits grace the present volume and form a valuable addition.

The remaining 225 pages are devoted to the scientific addresses and papers read at the congress. The address of Poincaré has already appeared in the *Acta Mathematica*, but is also given here. Those of Klein and Hurwitz, I have not seen in print elsewhere. I wish to call particular attention to Hurwitz's paper. This is without doubt the most careful and broadly conceived sketch at hand of the recent developments, in a certain well defined field, of the general theory of functions. Over 130 references to original sources are given in the course of the memoir.

The volume contains either in full or in abstract about thirty papers read in the various sections. I note only a few of general interest: H. Weber, Genera of algebraic bodies; E. Schröder, Pasigraphy; L. Stickelberger, Discriminants of algebraic numbers; F. Meyer, Algorithms similar to continued fractions; T. Reye, Quadratic complexes; A. Stodola, Relation of technical sciences to mathematics; H. Zeuthen, Barrow's method of inverse tangents; G. Eneström, Mathematical bibliography; G. Loria, Historical sketch of plane curves.

Some of the papers have appeared elsewhere and are given here only as abstracts. All will regret that Picard's paper on algebraic functions of several variables is given only in the briefest form. The first volume of the Fonctions algébriques de deux Variables which has since appeared will not console us for the loss of this brilliant aperçu which was listened to with intense interest by almost the whole congress.

Mathematicians will be grateful to Professor Rudio for this very complete and attractive report. The book contains so much of general interest that it will be welcome to all.

JAMES PIERPONT.

The Mathematical Theory of the Top. Lectures delivered on the occasion of the Sesquicentennial Celebration of Princeton University. By FELIX KLEIN, Professor of Mathematics in the University of Göttingen. New York, Charles Scribner's Sons, 1897. 12mo, iv + 74 pp.

THESE lectures "were reported and prepared in manuscript form by Professor H. B. Fine, of Princeton University, and the manuscript was revised by Professor Klein." The twelve figures in the text, some of which required great delicacy in construction, were drawn by Professor H. S. S. Smith. The subject matter of the lectures is divided into four parts: the geometric introduction; the dynamic relations; the general analytic discussion when the independent variable is complex; the toy top with moving point.

First of all, the double system of coördinates used in considering the problem of the rotation of a body about an axis with one point fixed is considered, and the attempt is made "to throw a method already well known into the most convenient form for application to mechanics." As the leading expounder today of the ideas of Riemann it is fitting that Professor Klein should use the transformations of the system derived from the stereographic projection of the points of the sphere, rather than the system employed by Euler, or the system which may be designated as the quaternion system. In the beginning the transformations are expressed much more symmetrically in these latter systems, but the problem is not one which is symmetrical, since the body rotates about one axis. And it would therefore seem (as the result shows) that the system of stereographic projection, which considers one line as distinguished from all others, should be superior. and that the expressions for this movement will have a kind of symmetry in this system. Moreover, such a system has been used with great power by French writers, notably Darboux. It is especially emphasized that, in this system of parameters, the parameters, as well as the magnitude ordinarily called the variable, are functions of the time. Since these parameters are complex numbers, and are functions of t, the analytic problem which

naturally emerges is to consider the rotation as depending upon a complex variable t. Certain of the properties dependent on this problem are stated in terms of the non-euclidean geometry, but "the non-euclidean geometry has no metaphysical significance here, but is used solely because it is a convenient method of grouping in geometric form relations which must otherwise remain hidden in formulas."

In considering the dynamical question, it is noted that the simpler movement under consideration is a Poinsot movement, and then the equations of Lagrange are introduced. It is only by the inversion of the integrals that the problems can be solved and this has been done by the methods of Jacobi. These considerations will be much simplified by the use of the parameters here introduced, and by the introduction of the Riemann method of conform representation, by means of which the parameters can be expressed as quotients of sigma functions which may be called multiplicative elliptic functions. In the same way the curve on the sphere about the point of the top, which may be called the polhode, is defined by a multiplicative elliptic function, as is also the herpolhode.

By means of the diagram of the stereographic projection of a Riemann surface on the sphere the functions of a complex t are represented, and the determination of this surface is explained by taking a single portion of this surface at one time.

Poinsot was the first to show that the motion of a top on a plane leads to hyperelliptic functions, but in the dynamic problem in hand Jacobi's method of inversion involves many difficulties of calculation. Professor Klein closed these lectures with the introduction of the expressions for the parameters in terms of the automorphic prime functions as these are better adapted to the purpose of inversion than the functions of Jacobi.

In an address before a special meeting of the AMERICAN MATHEMATICAL SOCIETY, held in Princeton, on Saturday, October 17, 1896, Professor Klein continued his contribution to the subject in considering the question of the stability of the sleeping top. An abstract of this address has been printed in this BULLETIN, January, 1897; 2d Series, vol. 3, pp. 129–132.

H. D. THOMPSON.

Politische Arithmetik oder die Arithmetik des täglichen Lebens. By MORITZ CANTOR. Leipzig, B. G. Teubner, 1898. Pp. ix + 136.

OUR author was fortunate in the choice of the title of his work, for the object of the book is to render intelligible to the lay reader the transactions of the business world which are based upon mathematics. Calculus is used to some extent but the concrete instances which follow and interpret the formulæ in most cases render the meaning clear to the average reader. To the American the space devoted to interest and speculation in nearly superfluous, but he finds of interest the chapter devoted to the description of the various systems of lotteries that have played so large a rôle on the continent. I know of no work in which the theory of probabilities and the formation of life tables are more clearly and concisely developed. We look in vain for a demonstration of the economic loss arising from gambling which is so forcibly put in the writings of Daniel Bernoulli, but in a treatment from a purely mathematical standpoint, perhaps the author has done well to leave the ethical problems untouched, and not to attempt to walk on that slippery ground that separates legitimate from illegitimate speculation. With a clear knowledge of this book the reader will be better prepared to appreciate the work being done by such writers as Pareto and F. Y. Edgeworth. Within the limits set to himself Cantor has succeeded admirably, and the average reader will find the financial page of the daily paper more intelligible from a careful perusal of his little book.

WM. B. BAILEY.

Introduzione alla Economia Matematica. Dei PROFESSORI F. VIRGILII e C. GARIBALDI. Milan, M. Hoepli, 1899. 18mo, pp. x + 210.

THE use of mathematical method is rapidly increasing among economists, so that a training in it is now essential in order to understand much of the best modern work. To supply such training in the use of symbolic argument in this and allied fields is the object of the present work. It is interesting to Americans to note that the first book in this line, slightly briefer than this, is by Professor I. Fisher, of Yale. When reviewed in Italy it received high commendation, and the volume in hand is an effort to do for the Italian scholar the same thing that was attempted for English readers by Professor Fisher.

The chief interest in the book lies in the historico-critical Introduction of 69 pages, in which the principal economists who have employed the mathematical method, and their leading works are noted—a growing list, in which English and American authors are rightfully preëminent. It closes with an analysis of the place of this method in economics as a quantitative science, and quotes many of the instances in which the errors of the older economists have been cleared away, and other cases in which truths undreamt of and unattainable under cruder conditions have been won by it. The authors long for some Laplace to unite into one system the rather scattered materials now at hand, and forecast a bright future for the science which is as yet the youngest of the mathematical family. A bibliography of a dozen pages (condensed from one by Professor I. Fisher in Cournot's Mathematical principles of the theory of wealth, Macmillan, '97) serves to indicate, forcibly though incompletely, the increased application of mathematical method in recent years.

The work proper is divided into three books: I, Elementary algebra; II, Elements of trigonometry and analytic geometry; III, Elements of theory of functions of real variables—occupying 40, 21 and 69 pages respectively. In book I are short summaries of elementary operations; of equations of the first degree and of the second degree with one unknown; of logarithms; of combinations; and of probabilities. Book II has two short chapters—on distance and angles, and on Cartesian coördinates. With book III come those portions which are most generally useful to economists. Part I—Differential calculus—gives preliminary ideas on functions and limits, derivatives, differentials, functions of many variables, maxima and minima. The last chapter, and the two on integrals which follow, are perhaps of the greatest service.

The book is a model of clearness and discretion. Each subject is treated simply and with a view to its application. Many of the fundamental problems of economics are taken up in connection with the development of mathematical theorems, and the whole forms a body of material which ought to be of the greatest service to economists. It should help to do away with the necessity hitherto felt by nearly every one of them who uses mathematics, of defending his practice, and of writing a text book of mathematics in his opening chapters. It might well serve as a text book in any case where only the elements of calculus are desired, and application is to be made at once to economics, psychology or the like.

ON ELLIPTIC FUNCTIONS.

BY PROFESSOR JAMES PIERPONT.

How simply and naturally to present the theory of elliptic functions is an embarrassing question. Since the functions of Jacobi and Weierstrass are both used by eminent mathematicians it is necessary to treat them both. This is usually done by giving one of the two classes the preference and deducing the properties of the other as corollaries. Thus in the works of Thomae and Weber the functions of Jacobi are given the preference; in the works of Halphen, Jordan, and Tannery and Molk it is the functions of Weierstrass.

I do not believe this course tends to show to best advantage the peculiar nature of each class of functions nor to show up their similarities and differences.

In my opinion the true foundation of Jacobi's theory is the *thetas*, of Weierstrass's the *sigmas*. To deduce the properties of the functions of Jacobi as Jordan has done in the new edition of his excellent Cours seems to me to rob the reader of half the beauties of this theory. The same is true of the functions of Weierstrass when deduced from those of Jacobi.

Another feature we should look for in a satisfactory presentation of the theory is one too often overlooked. We should demand that, since the sigmas and thetas are the fundamental elements of our theory, these should not be brought on to the scene like a kind of *deus ex machina* but that they appear as necessary elements of our reasoning. The introduction of the sigmas is made easy and natural by virtue of Weierstrass's factor theorem. To introduce the thetas naturally is less easy. Weber has shown with great success how the properties of the functions of Jacobi may be studied by means of the T functions, that is one valued transcendental functions which satisfy the relations

$$\begin{aligned} T(u+2\omega_1) &= e^{-\pi i[2a_1(u+\omega_1)+b_1]}T(u) \\ T(u+2\omega_2) &= e^{-\pi i[2a_2(u+\omega_2)+b_2]}T(u) \end{aligned} \tag{1}$$

where
$$R\left(\frac{\omega_2}{\omega_1}\right) > 0 \tag{2}$$

and
$$2a_2\omega_1 - 2a_1\omega_2 = m, \quad \text{an integer.} \tag{3}$$

The motives Weber gives for introducing such a function are so meager that the reader is left in wonder how they were ever thought of. He is also held in painful suspense for twenty-two pages whether such functions, whose existence is postulated, really exist.

Now the T functions, containing the thetas and sigmas as special cases, afford an excellent standpoint from which the functions of Jacobi and Weierstrass may be viewed, if we can only arrive at them in a natural manner, by the aid of a few à priori considerations. This I suggest doing in the following manner by starting from the notion of periodicity.

I would begin by remarking that having examples of one valued functions which are *simply* periodic and without essentially singular points in the finite part of the plane, it is natural to inquire after such functions that have more than one period. We show easily that such functions can be at most doubly periodic and that their periods, say $2\omega_1$, $2\omega_2$, cannot have a real ratio. For brevity call such functions elliptic. That they exist is shown at once by simple examples, as

$$f(u) = \sum_{\tilde{\omega}} \frac{1}{(u-\tilde{\omega})^m} \qquad (\tilde{\omega} = 2\lambda_1\omega_1 + 2\lambda_2\omega_2, \quad m > 2.)$$

Next we would show that an elliptic function takes on every value in a parallelogram of periods $P = (2\omega_1, 2\omega_2)$ as often, say m times, as it becomes infinite, and that $m > 1$. This we call the order of the function. Take now the most general elliptic function $\varphi_m(u)$ of order m. Let its zeros and poles in P be respectively

$$a_1, a_2 \cdots a_m,$$

$$b_1, b_2 \cdots b_m.$$

Let the totality of numbers $\equiv a_\kappa$ *modd.* $2\omega_1, 2\omega_2$ be represented by α_κ while those congruent to b_κ may be β_κ. Call α the totality of α_κ, and β the totality of β_κ.

Then since

$$\Sigma \frac{1}{|\alpha|^3}, \qquad \Sigma \frac{1}{|\beta|^3}$$

converge, Weierstrass's factor theorem gives at once

$$\varphi_m(u) = e^{g(u)} \frac{\prod_{\alpha} \left(1 - \frac{u}{\alpha}\right) e^{\frac{u}{\alpha} + \frac{1}{2}\left(\frac{u}{\alpha}\right)^2}}{\prod_{\beta} \left(1 - \frac{u}{\beta}\right) e^{\frac{u}{\beta} + \frac{1}{2}\left(\frac{u}{\beta}\right)^2}}.$$

The products $\prod_{\alpha}$ and $\prod_{\beta}$, being absolutely convergent, can be broken up into partial products so that

$$\varphi_m(u) = e^{g(u)} \frac{\prod_{\kappa=1}^{m} \prod_{a_\kappa} \left(1 - \frac{u}{a_\kappa}\right) e^{\frac{u}{a_\kappa} + \frac{1}{2}\left(\frac{u}{a_\kappa}\right)^2}}{\prod_{\kappa=1}^{m} \prod_{\beta_\kappa} \left(1 - \frac{u}{\beta_\kappa}\right) e^{\frac{u}{a_\kappa} + \frac{1}{2}\left(\frac{u}{a_\kappa}\right)^2}},$$

and now, making use of the fact that the second logarithmic derivatives of $\prod_{a_\kappa}, \prod_{\beta_\kappa}$ are doubly periodic, we show in the customary manner that $g''(u)$ is a constant and therefore that $g(u)$ is a polynomial of second degree while $\prod_{a_\kappa}, \prod_{\beta_\kappa}$ are of the form

$$\tau(u \mid a, p, q, r) = e^{pu + qu + ru^2} \sigma(u - a).$$

Hence the most general elliptic function of order m is the quotient of two integral transcendental functions of the type

$$T^{(m)}(u) = \prod_{\kappa}^{1, m} \tau(u \mid a_\kappa, p_\kappa, q_\kappa, r_\kappa). \tag{4}$$

These functions satisfy the conditions (1), (2), (3); and conversely, a one valued integral transcendental function satisfying these conditions is a function of the type (4). We have thus reached our goal.

YALE UNIVERSITY,
April, 1899.

NOTES.

In connection with the celebration of the decennial anniversary of the founding of Clark University, to be held July 5–8, several courses of lectures have been arranged to be delivered by distinguished foreign scientists. Among others a course of three lectures on a subject to be announced will be given, July 5–7, by Professor É. Picard, of the University of Paris, and a course of four lectures on the principles and fundamental equations of mechanics, July 5–8, by Professor L. Boltzmann, of the University of Vienna. The hours of the various courses will be so arranged as not to conflict. Tickets for admission may be obtained, without charge, on application to Mr. L. N. Wilson, Clark University.

The sixty-ninth annual meeting of the British association for the advancement of science will be held at Dover beginning on Wednesday, September 13th. Professors Michael Foster and J. H. Poynting are the respective presidents of the association and of the section devoted to mathematical and physical science.

The French association will hold its annual meeting at Boulogne at the same time. Members of the French association will visit Dover on Saturday, September 16th, and members of the British association will visit Boulogne on Thursday, September 21st.

The Jubilee of Sir George Gabriel Stokes, Lucasian professor of mathematics at the University of Cambridge since 1849, was celebrated at Cambridge June 1st and 2d. The University of Cambridge presented an address and a commemorative gold medal to Professor Stokes, and the Academy of Sciences of Paris conferred the Arago medal upon him; and the Berlin Academy of Sciences transferred his name from the roll of correspondents to that of foreign members. Delegates presenting addresses were present from seventy-five universities and scientific societies. The honorary degree of doctor in science was conferred upon Professors M. A. Cornu, of Paris, J. G. Darboux, of Paris; A. A. Michelson, of Chicago; M. G. Mittag-Leffler, of Stockholm; G. H. Quincke, of Heidelberg and W. Voigt, of Göttingen.

The Redo lecture was delivered June 1st by Professor M. A. Cornu, on "The theory of luminous waves; its influence on modern physics."

AMONG those proposed for election as fellows of the London Royal Society is H. W. LLOYD TANNER, professor of mathematics at Cardiff College, South Wales.

PROFESSOR GIBSON, in a review of the third volume of M. Cantor's Vorlesungen über Geschichte der Mathematik, which appears in the Proceedings of the Edinburgh Mathematical Society, vol. 17, gives some very interesting details of the controversy which was caused by Bishop Berkeley's criticism of the doctrine of fluxions. The part played in this controversy by Benjamin Robins (1707–1751) appears to have been overlooked. One has associated Robins only with the Ballistic Pendulum and with a work on Gunnery which Euler translated into German (Routh, Rigid Dynamics, ch. iii.). But, to quote Professor Gibson, "to Robins, more than to Maclaurin, I think is due the credit of expounding in systematic and consistent form the fundamental conception of a limit, and of freeing Newton's statement from the ambiguities which gave plausibility to Berkeley's attack."

THE number of *Nature* for May 4, 1899, contains a sketch of the scientific work of Professor SIMON NEWCOMB by Professor M. LOEWY, director of the Paris Observatory. The sketch is also reprinted in the Johns Hopkins University Circular for June, 1899.

THE library of the late Dr. HERMANN SCHAPIRA is offered for sale by Gustav Fock of Leipzig.

CORNELL UNIVERSITY. During the academic year 1899–1900, the following advanced courses will be offered by the Department of Mathematics, each course occupying either two or three hours a week as indicated, and continuing through the year :—By Professor WAIT : Advanced analytic geometry, (a) plane, three hours, (b) solid, two hours ; Advanced differential calculus, three hours.—By Professor JONES : Higher algebra and trigonometry, three hours ; Probabilities, etc., two hours.—By Professor MCMAHON : Theoretical mechanics, two hours ; Quaternions, two hours ; Potential and spherical harmonics, two hours ; Mathematical theory of sound, two hours.—by Professor TANNER : Algebraic invariants, two hours ; German readings, two hours.—By Dr. MURRAY : Differential equations, three hours ; Finite differences, etc., two hours.—By Dr. HUTCHINSON : Advanced integral calculus, two hours ; Theta and abelian functions ; three hours.—By Dr. SNYDER : Projec-

tive geometry, three hours: Algebraic curves, two hours; General function theory, three hours; Line geometry, two hours.—By Dr. MILLER: Theory of groups of an infinite order, three hours; Theory of numbers, two hours.

The Oliver mathematical club will hold fortnightly conferences.

DR. EDWIN S. CRAWLEY, assistant professor of mathematics at the University of Pennsylvania has been promoted to a full professorship, succeeding to the chair held for many years by the late E. Otis Kendall.

Dr. J. V. WESTFALL, honorary fellow in Cornell University, has been appointed instructor in mathematics in the Iowa State University.

H. M. MACDONALD, M.A., of Clare College, has been appointed a University lecturer in mathematics in Cambridge University.

IN Cambridge University the Adams prize for 1899 is awarded to Dr. J. LARMOR and Mr. G. T. WALKER. The Smith's prizes are adjudged to W. H. AUSTIN for an essay on "The motion of a symmetrical top on a smooth horizontal plane," and to G. W. WALKER for an essay "On the scattering of light by small slightly conducting particles."

*NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

BIANCHI (L.). Lezioni sulla teoria delle funzioni di variabile complessa e delle funzioni ellittiche. Parte II. Pisa, 1899. 8vo. 480 pp. Fr. 12.50

BÖTTCHER (J. E.). Beiträge zur Theorie der Iterationsrechnung. Leipzig, 1898. 8vo. 78 pp. M. 2.00

DAHLBO (J.). Uppränning till Mathematikens Historia i Finnland fran äldsta tider till storia ofreden. Helsingfors, 1897. 8vo. 196 pp. M. 3.20

D'ARCAIS (F.). Corso di calcolo infinitesimale. Volume I. Seconda edition con aggiunte e modificazioni. Padua, Angelo Draghi, 1899. 8vo. 9 and 523 pp.

DÖRRIE (H.). Das quadratische Reciprocitätsgesetz im quadratischen Zahlkörper mit der Classenzahl 1. [Diss.] Göttingen, Vandenhoeck & Ruprecht, 1899. 8vo. 75 pp. M. 2.40

FISCHER (E.). Ueber Potenzen mit imaginären Exponenten. Berlin, 1899. 4to. 25 pp. M. 1.50

FOURREY (E.). Récréations arithmétiques. Paris, 1899. 8vo. Fr. 4.00

GEIGENMÜLLER (R.). Elemente der höheren Mathematik zugleich als Sammlung von Beispielen und Aufgaben aus der analytischen Geometrie, algebraischen Analysis, Differential- und Integralrechnung. 2 Bände. Mittweida, Polytechnische Buchhandlung. 8vo. M. 12.00

HAMILTON (W. R.). Elements of quaternions. 2d edition by C. J. Joly. Volume I. London, Longmans, 1899. 4to. 21s.

JOLY (C. J.). See HAMILTON (W. R.).

LEWIN (M.). Ueber die Principien von Hamilton und Maupertuis. Heidelberg, 1898. 8vo. 49 pp. M. 2.00

LUDWIG (W.). Ueber die Ebenen, welche aus einer Fläche zweiten Grades einem gegebenen Kegelschnitt ähnliche Schnitte ausschneiden. Berlin, 1898. 8vo. 74 pp. M. 1.80

MANNHEIMER (N.). Untersuchungen über die Möglichkeit algebraischer Differentialgleichungen für additiv- und multiplicatorisch- periodische Functionen zweiter Gattung. Heidelberg, 1898. 8vo. 30 pp. M. 1.50

MICHELSEN (P.). Die bestimmten algebraischen Gleichungen des ersten bis vierten Grades. Nebst einem Anhang: Unbestimmte Gleichungen. Zweite Auflage. Hannover, 1899. 8vo. 8 and 306 pp. M. 4.00

PATERNO (F. P.). Sopra un notevole sistema isogonico di proiezioni parallele. Palermo, 1899. 4to. 8 pp. F. 1.75

PEARSON (K.). Mathematical contributions to the theory of evolution. Part V. On the reconstruction of the stature of prehistoric races. London, 1899. 4to. 76 pp. 4s.

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*For the monthly list of New Publications contained in the volume herewith concluded, the BULLETIN is indebted to Professor J. L. MARKLEY, University of Michigan.

PINCHERLE (S.). A proposito di un recente teorema del Hadamard. Bologna, Gamberini e Parmeggiani, 1899. 8vo. 10 pp.

WEBER (H.). Lehrbuch der Algebra. Zweite Auflage, Band 2. Braunschweig, F. Vieweg & Sohn, 1899. 16 and 855 pp. M. 12.00

II. ELEMENTARY MATHEMATICS.

ATKINSON (E. H. DE V.). Text-book of practical solid geometry, etc., for the use of the royal military academy, Woolwich. London, Spon, 1899. 8vo. 124 pp. 7s. 6d.

BAUBEAU ET HATAT. Arithmétique. Tours, Deslis frères, 1899. 16mo. 224 pp.

BRIGGS (W.) and EDMONDSON (T. W.). Mensuration and spherical geometry. Including elements of geometry of rectilinear solids and sphere. 3d edition. London, Clive, 1899. 8vo. 172 pp. 3s. 6d.

ESTEL (B.). See SÄRCHINGER (E.).

F. M. G. Cours de géométrie élémentaire, suivi d'un recueil d'exercices proposés. Tours, Mame, 1899. 8vo. 12 and 525 pp.

FRÈRES DES ÉCOLES CHRÉTIENNES. Eléments d'algèbre, avec de nombreux exercices. 10e édition. Paris, Poussielgue, 1899. 16mo. 8 and 409 pp.

—— Exercices de calcul sur les quatre opérations fondamentales de l'arithmétique. Livre de l'élève. Paris, Poussielgue, 1899. 12mo. 8 and 72 pp.

—— Nouveau traité d'arithmétique décimale contenant toutes les opérations ordinaires du calcul, etc. Paris, Poussielgue, 1899. 18mo. 380 pp.

FRIAUCOURT (G.). Tables de logarithmes à six decimales pour les nombres et les lignes trigonométriques et tables de navigation. Paris, 1899. 8vo. Fr. 9.00

GUIDE to mathematics of intermediate examinations in arts and science of the University of London. 5th edition. London, Clive, 1899. 8vo. 102 pp. 2s.

—— to mathematics of the matriculation examinations of the University of London. 7th edition. London, Clive, 1899. 8vo. 148 pp. 1s. 6d.

GLINZER (E.). Lehrbuch der Elementar-Geometrie. Siebente Auflage. Theil I. Planimetrie. Mit einer Sammlung von 300 Aufgaben. Dresden, 1899. 8vo. 5 and 120 pp. M. 1.80

HATAT. See BAUBEAU.

HELLSTRÖM (C. G.). Planimetriska, trigonometriska och stereometriska Problem. 2. upplaga. Stockholm, 1899. 8vo. 90 pp. M. 1.30.

MATHEMATICS, matriculation, model answers. From June, 1893, to January, 1899. University Tutorial Series. London, Clive. 8vo. 150 pp. 2s.

MEIFUSS (I. F.). Graded arithmetical problems for elementary schools: based on E. Hentschel's works. Pupil's revised edition. Parts 1, 2, 4. St. Louis, B. Herder, 1899. 16mo. 32 pp. each. $.05

MORABITO (G.). Memoria per l'equazione di secondo grado. Gerace Marina, Vincenzo Serafino e figlio, 1898. 8vo. 16 pp.

PLANT (E. C.). Geometrical drawing for army and navy candidates and public school classes. Volume I., Practical plane geometry. Appendix on construction of scales. London, Macmillan, 1899. 4to. 200 pp. 7s. 6d.

SÄBCHINGER (E.) und ESTEL (B.). Aufgabensammlung für den Rechenunterricht in den Unterklassen der Gymnasien, Realgymnasien, and Realschulen. In 3 Heften. Zweite, verbesserte Auflage. Leipzig, Teubner, 1899. 8vo.

TROPFKE (J.). Erstmaliges Aufbreten der einzelnen Bestandteile unserer Schulmathematik. Teil I. Berlin, 1899. 4to. 27 pp. M. 1.00

WOOD (W. J.). Graduated test papers in elementary mathematics, modelled on Subject V., Stage I., of science and art department examination. London, Macmillan, 1899. 8vo. 1s.

III. APPLIED MATHEMATICS.

BAYER (A.). Theorie der Transformationen für dreiphasige Wechselströme. Zürich, 1898. 8vo. 106 pp. M. 3.00

BOULVIN (J.). Cours de mécanique appliquée aux machines proposé à l'Ecole spéciale du génie civil de Gand. 8e fascicule: appareils de levage; transmission du travail à distance. Paris, Bernard et Cie, 1899. 8vo. 30 and 253 pp.

CHOURA (J.). Leitfaden für den Unterricht in der darstellenden Geometrie an der k. k. Militär- Oberrealschule. Vienna, 1899. 8vo. 8 and 301 pp. M. 4.00

GARIBALDI (G.). See VIRGILII (F.).

GOLDSTEIN (J.). Die Statistik und ihre Bedeutung für das moderne Gesellschaftsleben. Munich, 1899. 8vo. 33 pp. M. 0.80

GOODMAN (J.). Mechanics applied to engineering. London, Longmans, 1899. 8vo. 614 pp. 7s. 6d.

HEAVISIDE (O.). Electromagnetic Theory. Volume II. London. "Electrician" Office, 1899. 8vo. 560 pp. 12s. 6d.

LAUENSTEIN (R.). Die graphische Statik. Fünfte Auflage. Stuttgart, 1899. 8vo. 240 pp. M. 5.40

LEBON (E.). Géométrie descriptive. 7e édition (2 volumes) Vol. I. Classe de seconde. Paris, 1899. 8vo. 150 pp. Fr. 3.50

LEECHENHORST (R. P. v.). Kartenprojectionen im allgemeinen und perspectivische Kartenprojectionen im besonderen. Theil II. Elbogen, 1898. 8vo. 31 pp. M. 2 00

LÖWENBERG (C. G. B. v.). Glückspiele. Eine philanthropisch-mathematische Studie. Munich, 1899. 8vo. 62 pp. M. 2.00

MOLINARI (T.). Il congegno matematico delle assicurazioni sulla vita spiegato elementarmente. Roma, 1899. 8vo. 220 pp. Fr. 7.00

SHEPPARD (W. F.). On the application of the theory of error to cases of normal distribution and normal correlation. London, 1899. 4to. 67 pp. 4s.

VIRGILII (F.) e GARIBALDI (G.). Introduzione alla economia matematica. Milan, 1899. 12mo. 222 pp. Fr. 2.00

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READ BEFORE THE AMERICAN MATHEMATICAL SOCIETY AND SUBSEQUENTLY PUBLISHED, INCLUDING REFERENCES TO THE PLACES OF THEIR PUBLICATION.

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—— On Singular Points of Linear Differential Equations with Real Coefficients. Read Oct. 29, 1898. *Bulletin of the American Mathematical Society*, vol. 5, No. 6, pp. 275–281; March, 1899.

—— An Elementary Proof that Bessel's Functions of the Zeroth Order Have an Infinite Number of Real Roots. Read Feb. 25, 1899. *Bulletin of the American Mathematical Society*, vol. 5, No. 8, pp. 385–388; May, 1899.

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—— Sur les Théorèmes de Green et de Cauchy. Read Dec. 28, 1898. *Comptes Rendus*, vol. 127, No. 10, pp. 604–606; March 6, 1899.

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—— The Largest Linear Homogeneous Group with an Invariant Pfaffian. Read Oct. 29, 1898. *Bulletin of the American Mathematical Society*, vol. 5, No. 7, pp. 338–342; Apr., 1899.

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—— Bestimmung aller ternären und quaternären Collineationsgruppen, welche mit symmetrischen und alternirenden Buchstabenvertauschungsgruppen holoedrisch isomorph sind. Read (Chicago) Dec. 30, 1897. *Mathematische Annalen*, vol. 51, No. 2, pp. 253–298; Oct., 1898.

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